Weak Chaos in the Plasma of a Fusion Device

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Ricerca Formazione Innovazione

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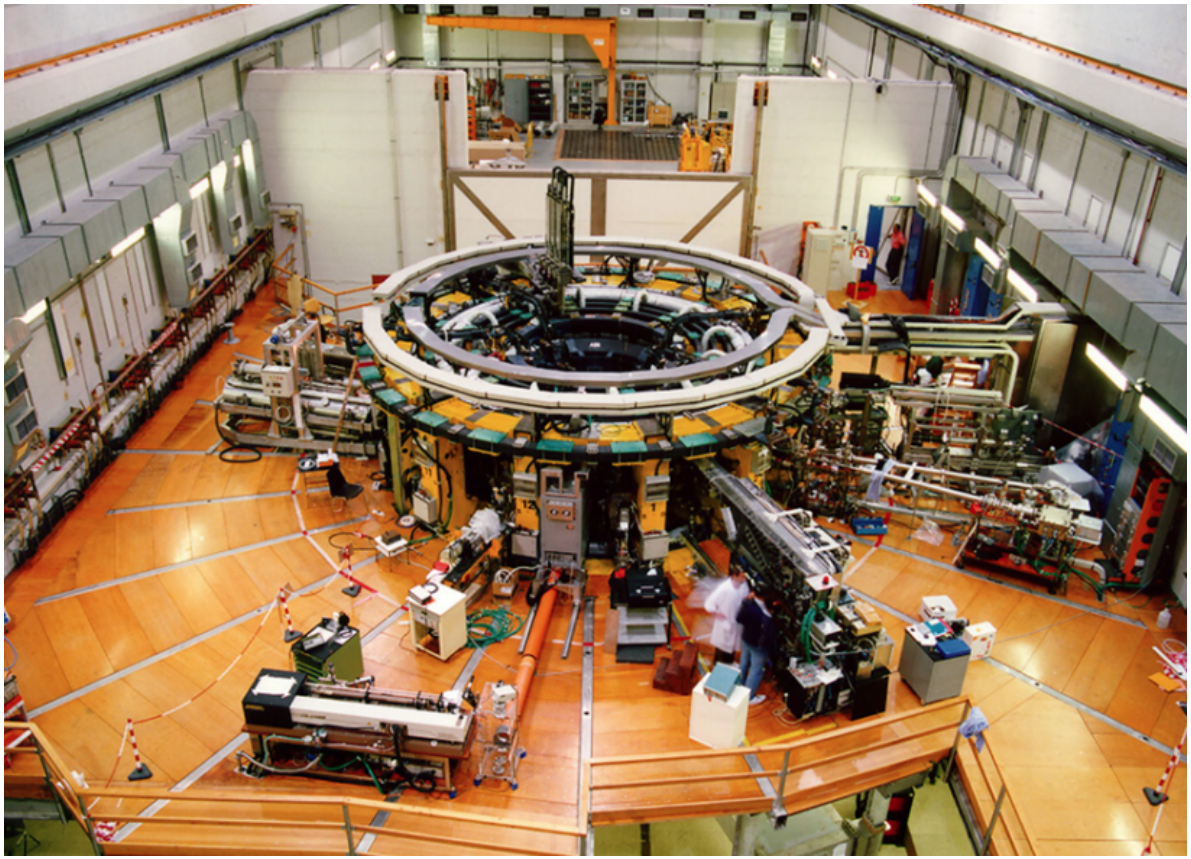
- Magnetic perturbations (spontaneous-magnetohydrodynamic, or imposed by external means) break KAM surfaces $\rightarrow$ islands
- Resonance of many magnetic islands bring in chaos
- Study and control of chaos is fundamental for nuclear fusion research $\rightarrow$ for example, for controlling Plasma-wall Interaction (PWI)

RFX group - Padova



RFX-mod$^{\alpha}$ [a]
reversed-field pinch (RFP) $\rightarrow R = 2m$, $a = 50cm$, plasma current $I_P \approx 0.3 \div 2$ MA
electron temperature $T_e \sim 0.2 - 1.2$ keV, density $n_e \sim 1 \div 9 \times 10^{19} m^{-3}$

[a] $\alpha = 0, 1, 2$

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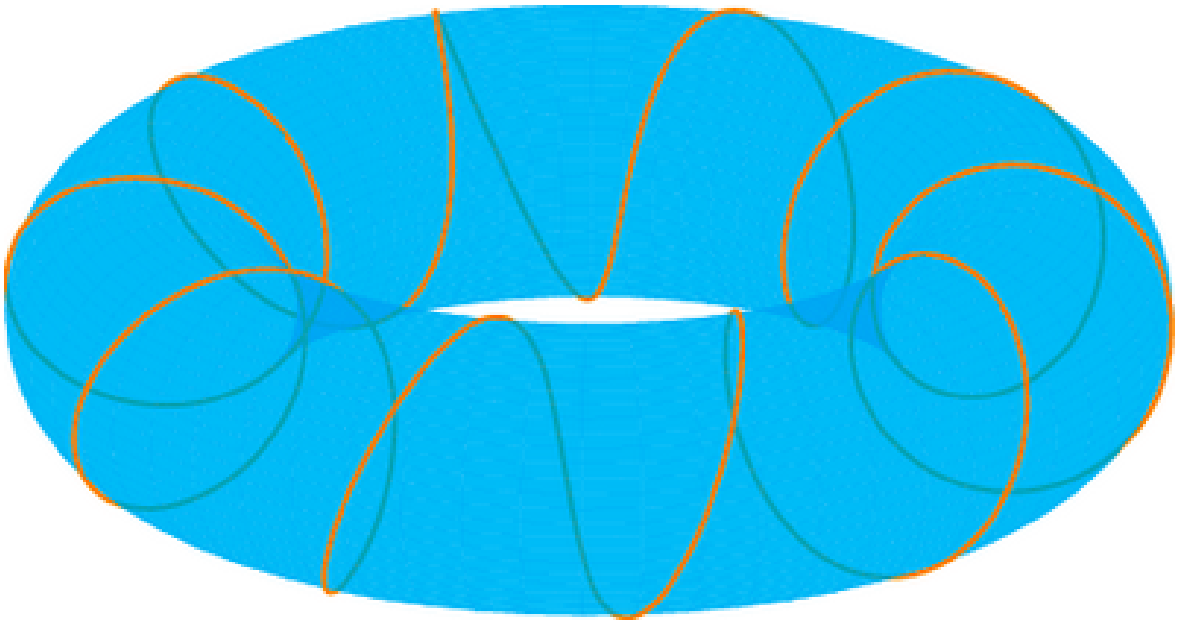


$$q = \frac{m}{n} = \frac{1}{10}$$

m, n are the poloidal & toroidal mode numbers

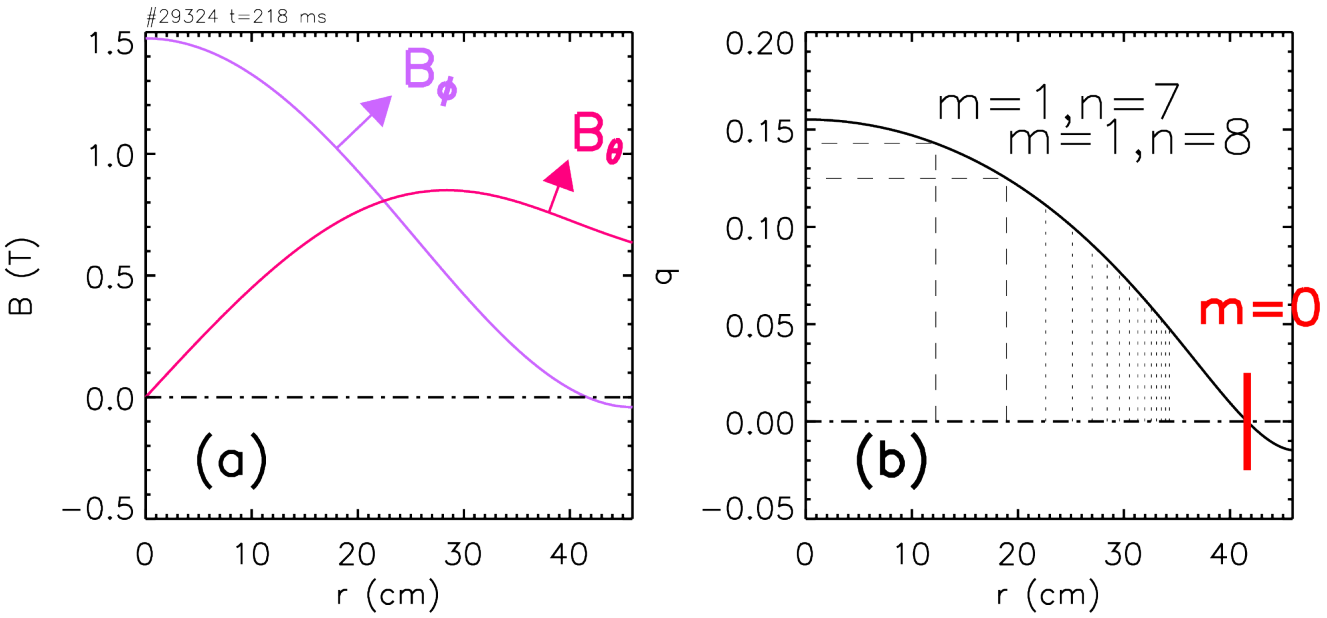
Definition of flux surface:

$$\begin{aligned} F(r,\theta,\zeta) &= const. \\ \vec{B}\cdot\nabla F &= 0 \end{aligned} \tag{1}$$

Magnetic field lines wind helically staying tangent to the flux surface

Toroidal flux surfaces $\equiv$ KAM tori

Equilibrium RFX - Padova



(a) **Toroidal** and **Poloidal** field $\vec{B}$
(b) Field helicity q, with first two resonances, q = 1/7 and q = 1/8
The other resonances, with $m = 1, n = 9, 10 \ldots$ are marked as vertical, dashed lines
Note the $m = 0$ **modes** resonating at the **reversal surface,** $q = 0$ which are typical of the RFP

Field-line Hamiltonian

The magnetic field line can be expressed in Hamiltonian form[1]
The time variable which appears in Hamilton equations is replaced by the poloidal angle θ (*small turn around the torus*)
Find the contravariant representation of the field $\vec{B}$

$$\vec{B} = \nabla\psi \times \nabla\theta - \nabla\psi_p \times \nabla\zeta \tag{2}$$

with ψ, ψ_p toroidal and poloidal flux, θ, ζ poloidal & toroidal angles
Identify

$$\begin{cases} \text{Hamiltonian} = & \psi \\ \text{Canonical conjugate momentum} = & \psi_p \\ \text{angle} = & \zeta \\ \text{time} = & \theta \end{cases} \tag{3}$$

[1] R.B. White, *The theory of toroidally confined plasmas*, Imperial College Press (2014), pp.10–13

Field-line Hamiltonian

Then the field-line Hamilton equations are:

$$\frac{\mathrm{d}\zeta}{\mathrm{d}\theta} = \frac{\partial\psi}{\partial\psi_p} = q \tag{2}$$

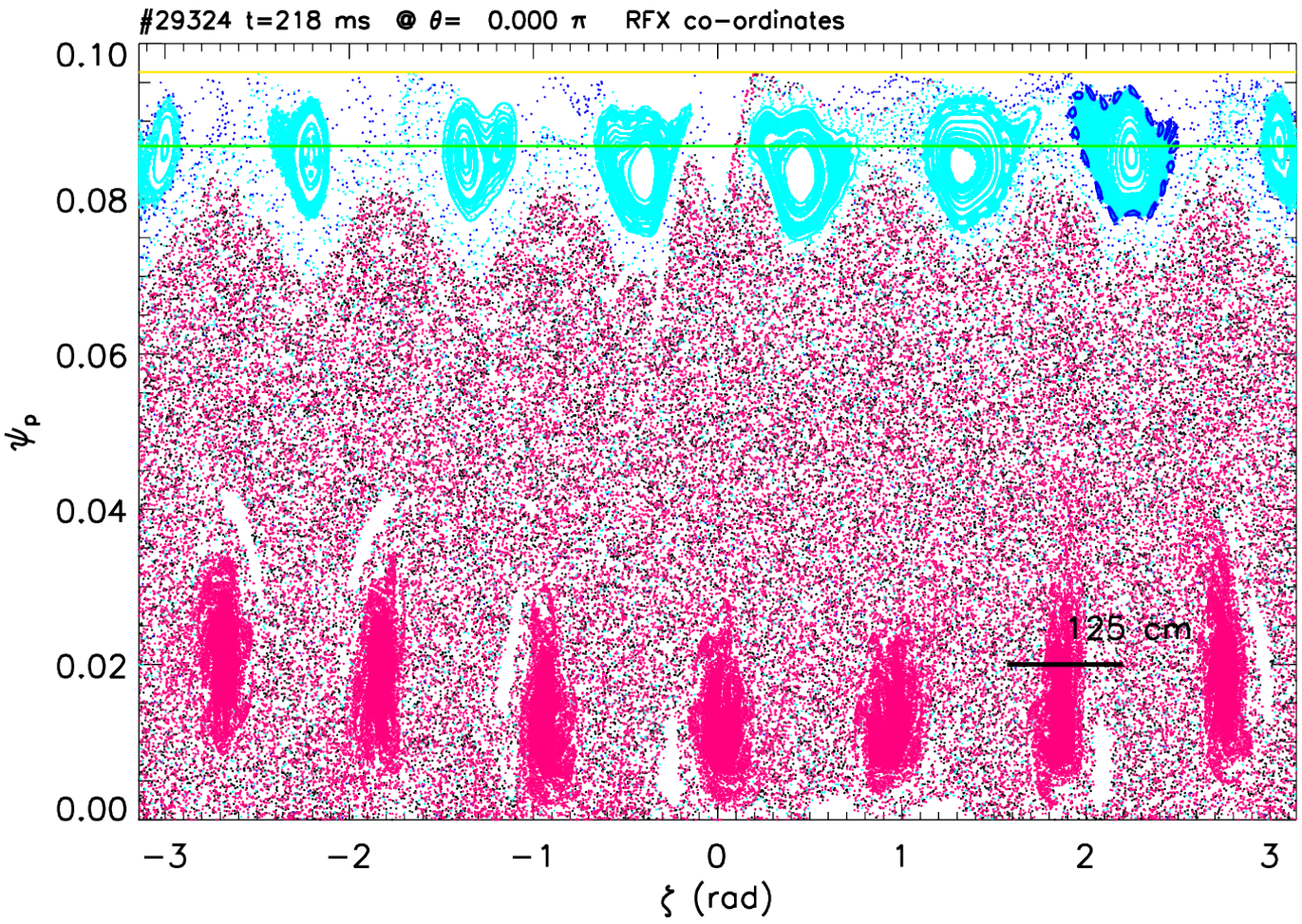
$$\frac{\mathrm{d}\psi_p}{\mathrm{d}\theta} = -\frac{\partial\psi}{\partial\zeta} \tag{3}$$

If the configuration is toroidally symmetric, $\frac{\partial\psi}{\partial\zeta} = 0$ and flux surfaces are conserved = intact KAM surfaces $\psi_p = const$. Perturbation can arise (spontaneously or induced on purpose) that break toroidal symmetry: the corresponding *perturbed* Hamiltonian can be expressed as

$$\psi(\psi_p, \theta, \zeta) = \int q \mathrm{d}\psi_p + \sum_{m,n} \frac{mg + nI}{n} \alpha_{m,n}(\psi_p) \sin(m\theta - n\zeta) \ . \tag{4}$$

which is in the form $H = H_0 + \alpha H_1$

RFP topology (code ORBIT)



Dominant mode is the $q = 1/7$ in the core, and $0/7$ in the plasma edge. In between, a chaotic sea determined by $m = 1, n > 7$ resonance overlapping.

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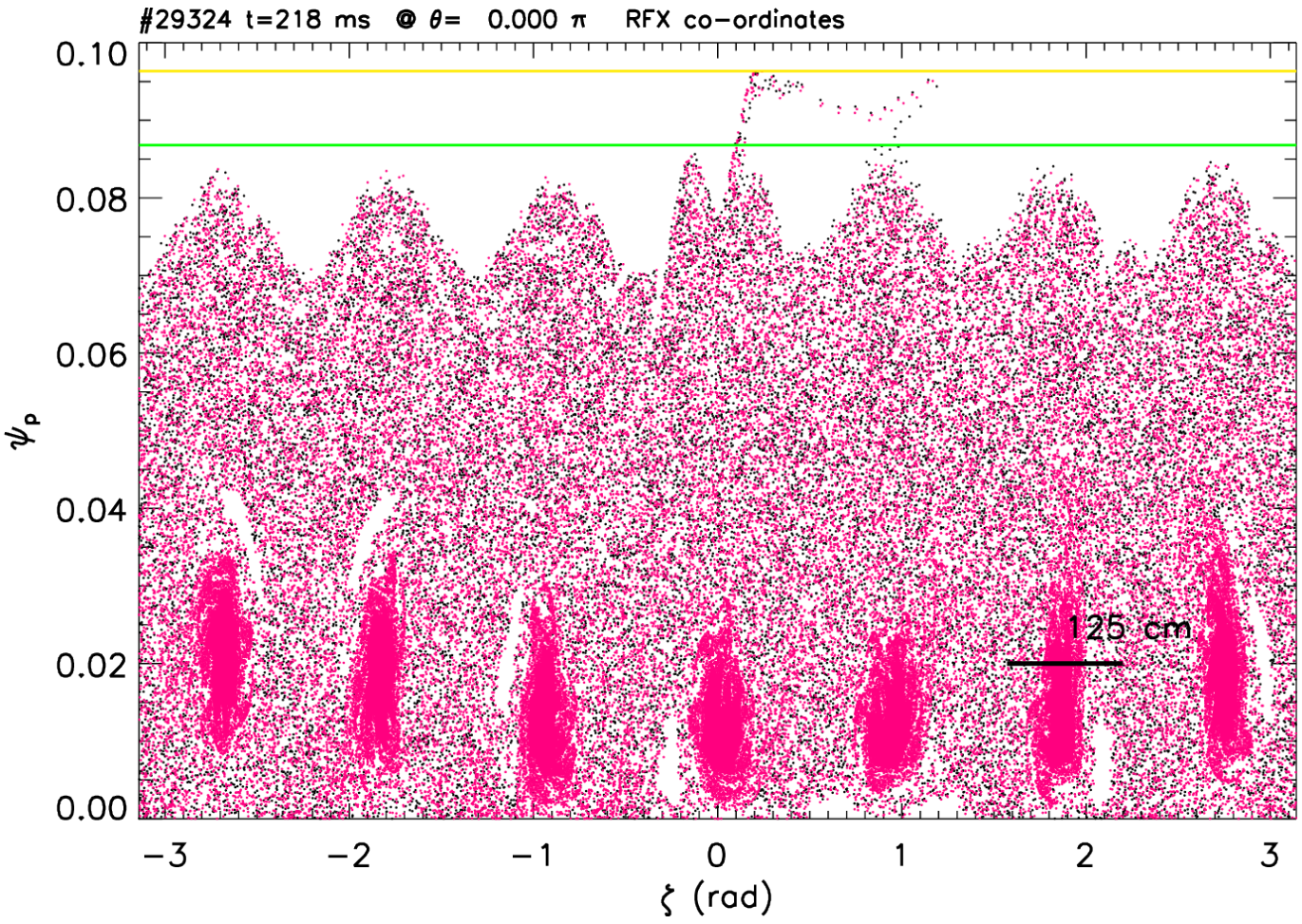


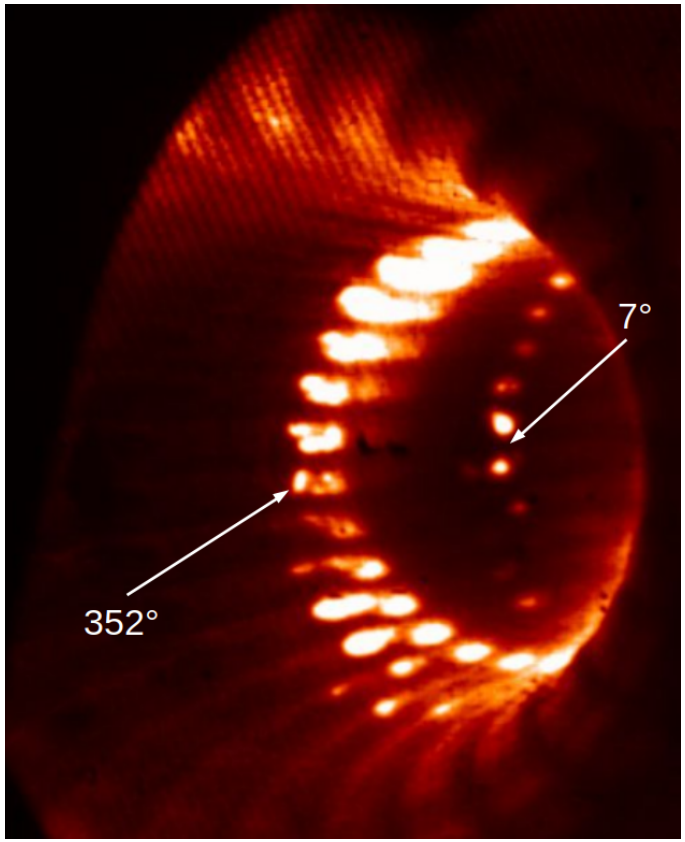
The chaotic sea touches the wall (yellow line) at $\zeta \approx 0$ in shot #29324 $\rightarrow$ phase locking of the $m = 1, n > 7$ modes (Locked Mode or LM)

Locked Mode and Plasma-Wall Interaction (PWI)

Locked modes are responsible for enhanced Plasma Wall Interaction (PWI) which induces overheating of the Plasma Facing Components, carbon sputtering, and radiation[1]
→ . . . all of these must be avoided in a fusion reactor!

Goals:

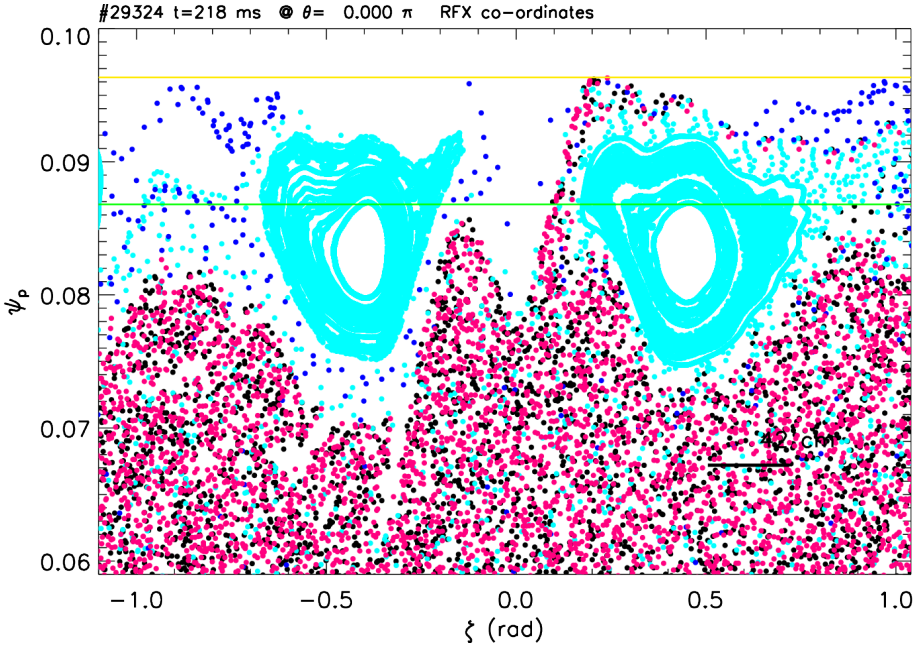
- Determine which modes are involved in the PWI event;
- Try to mitigate/avoid the LM and associated PWI.



[1] P.Scarin *et al*, Nuclear Fusion **59**, 086008 (2019)

Topology and connection lengths in the LM region

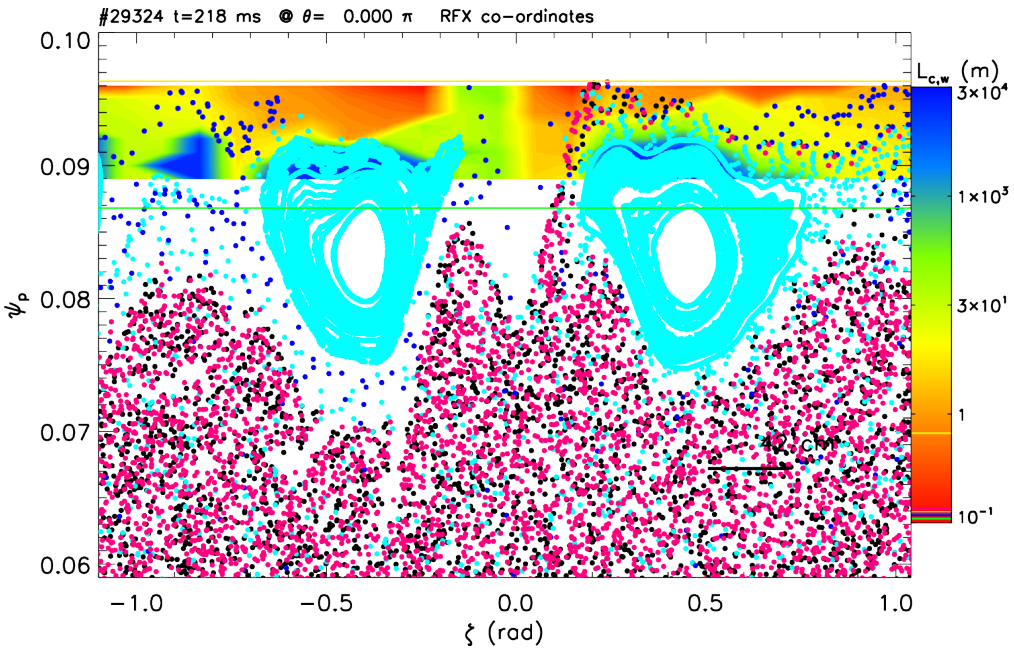
Topology in the region of the Locked Mode has two distinct symmetries:



- $m = 1$ chaotic sea which touches the wall;
- $m = 0$ islands.

Topology and connection lengths in the LM region

Metric to characterize the PWI: *connection length* $L_{c,w}$ to the wall[2]:



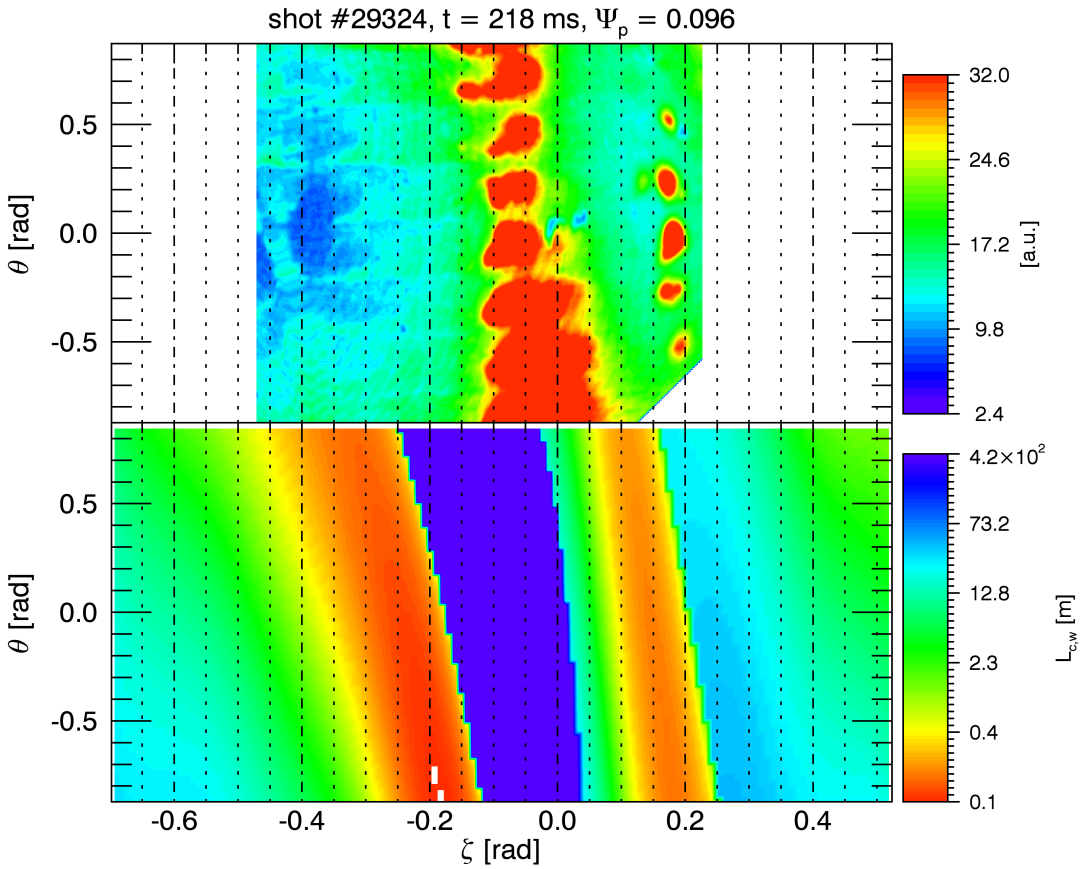
$$L_{c,w}(\psi_p, \theta, \zeta) = \int^{wall} \frac{B\, d\zeta}{\vec{B} \cdot \nabla \zeta} \quad (5)$$

Short $L_{c,w}$ = large plasma-wall interaction

[2] F. Nguyen and P. Ghendrih, Nuclear Fusion **37**, 743 (1997)

Comparison between connection length and camera images

The map of connection length and camera images of plasma-wall interaction qualitatively agree[3] :



- $L_{c,w}$ calculated at $\psi_p/\psi_w \sim 0.9$ on the (θ, ζ) plane shows two stripes

[3] Pasquale Porcu, Master's Thesis, University of Padova (2022)

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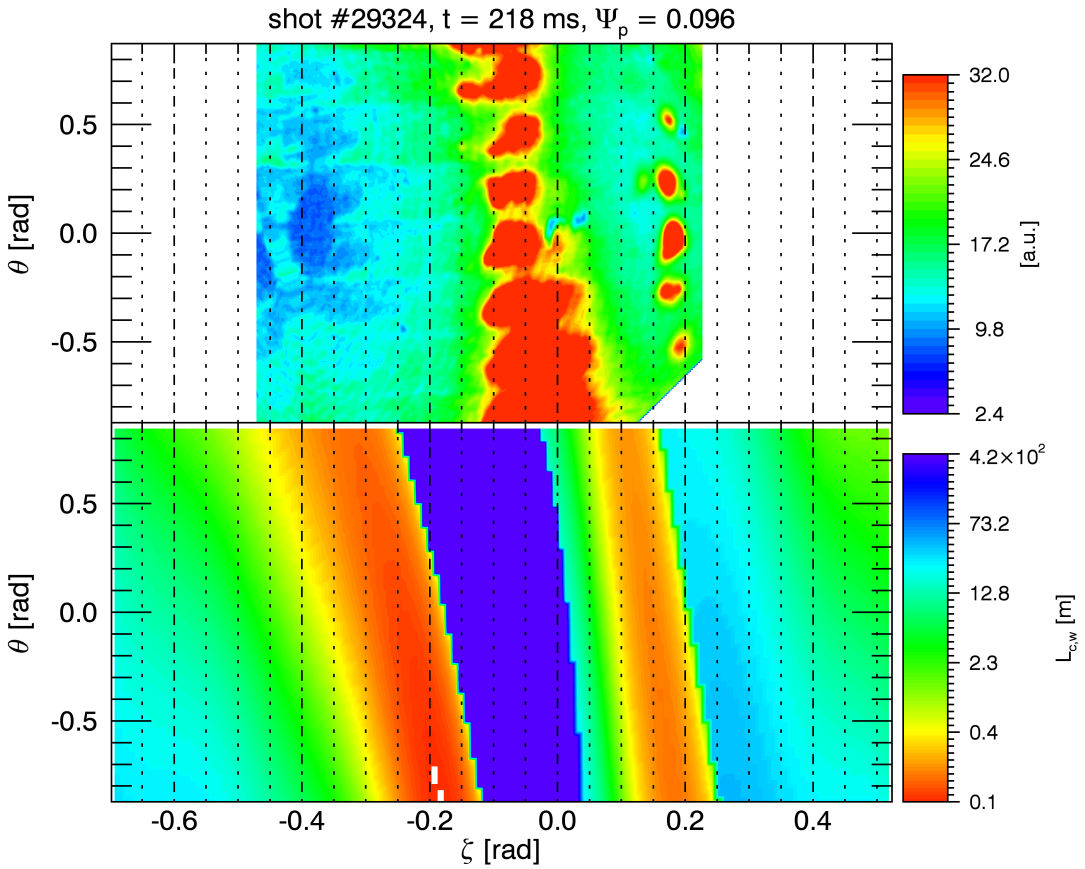

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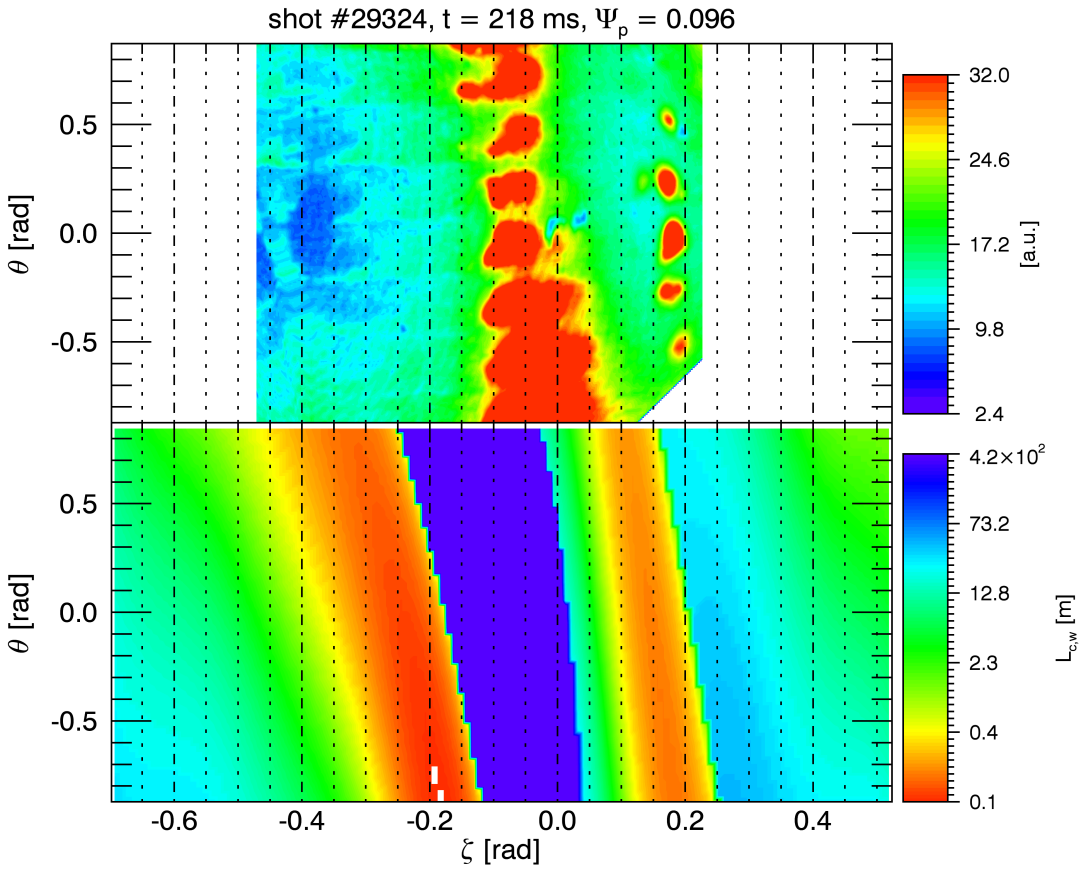

- $L_{c,w}$ calculated at $\psi_p/\psi_w \sim 0.9$ on the (θ, ζ) plane shows two stripes
- the **left, wide** one corresponds to the nearest $m = 0$ island
- the **right, narrow** one corresponds to the $m = 1$ locked mode

[3] Pasquale Porcu, Master's Thesis, University of Padova (2022)

Poincaré Recurrence Time

- We have seen that the connection length is a good indicator of the chaos topology and PWI in the plasma edge

[4] G. M. Zaslavsky, *Hamiltonian Chaos and Fractional Dynamics*, Oxford University Press, pp.173–186 (2004)

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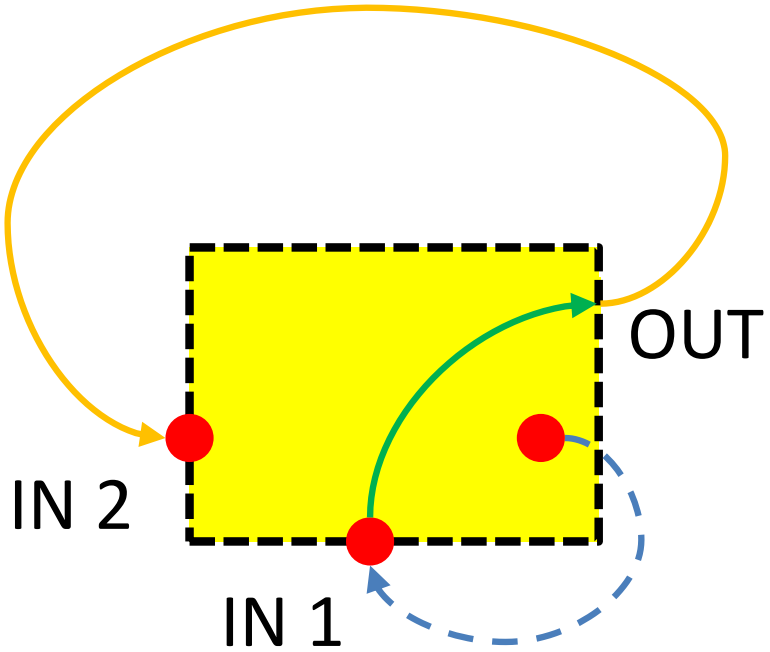

$$\begin{aligned}\tau^{(\mathrm{rec})} &= \tau^{(\mathrm{esc})} + \tau^{(\mathrm{ext})} \\ &= t_{OUT} - t_{IN1} + t_{IN2} - t_{OUT} \\ &= t_{IN2} - t_{IN1}\end{aligned} \tag{6}$$

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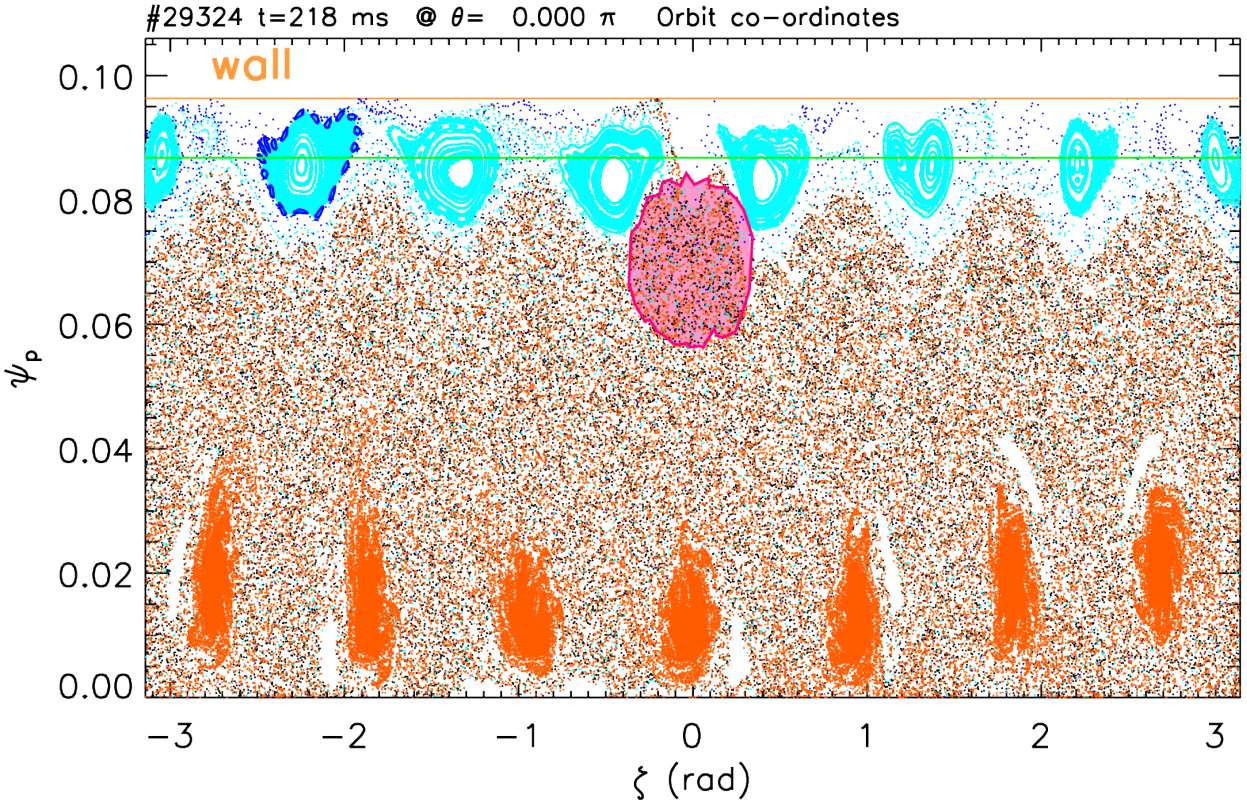
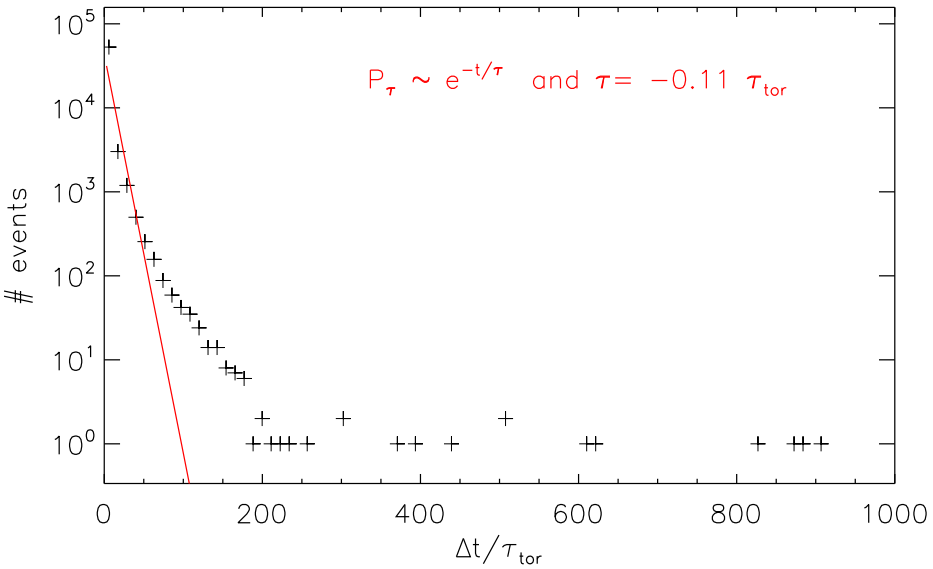
Poincaré Recurrence Time in RFX



Define an ellipsoid in (ψ_p, θ, ζ) near the LM at $\zeta \approx 0$.
Contrary to Boltzmann evaluation for a perfect gas, in a toroidal device $\tau^{(\mathrm{rec})} \lesssim 1000\,T$, with T the transit time of a particle (field line) on axis $\rightarrow$ it is not a very large time!

Statistics of recurrences

For each particle we record successive recurrence times in Eq. (6) and calculate the p.d.f. of recurrences



For short times we have an exponential law

$$P(t) = \frac{1}{\tau_{\mathrm{mix}}} \exp\left(\frac{-t}{\tau_{\mathrm{mix}}}\right) \tag{7}$$

where $\tau_{\mathrm{mix}} v_{th} = L_k$ is the Kolmogorov length

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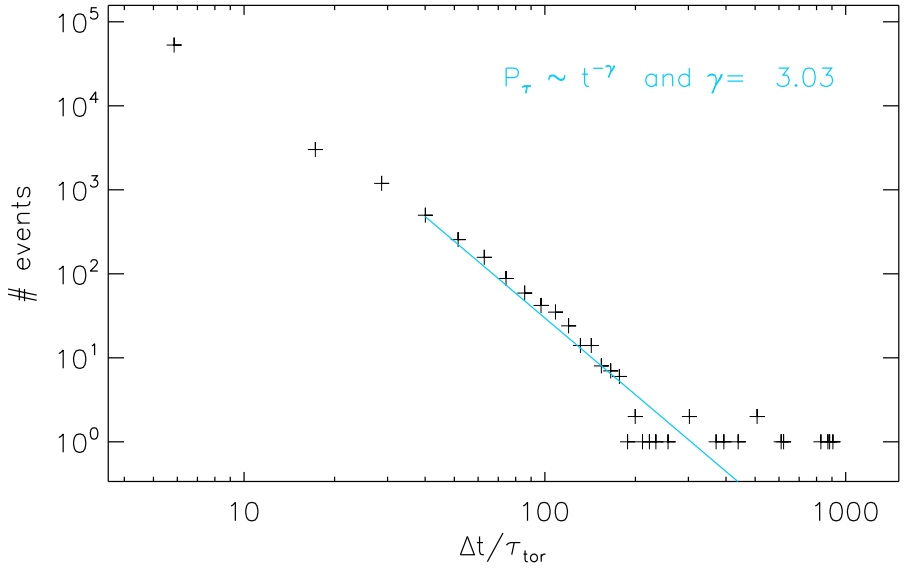
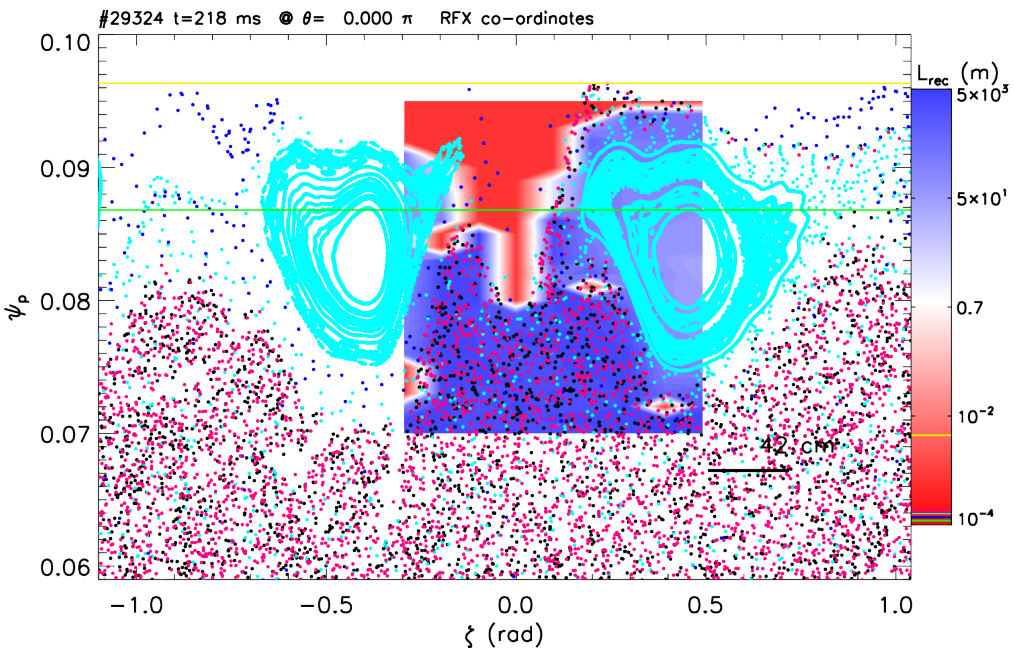


For longer times there is a power-law tail, $P(t) \sim t^{-\gamma}$ with $\gamma \sim 3$
$\rightarrow$ this tail an unambiguous proof of the existence of "dynamical traps" = islands and sticky regions with zero Lyapunov exponents[a]

[a] M. Veranda et al. Nuclear Fusion **60**, 016007 (2019)

Map of recurrences

Define the recurrence time as an average over the distribution[5]:



$$\tau_{\mathrm{rec}} = \frac{\int_0^{+\infty} t\, P(t)\mathrm{d}t}{\int_0^{+\infty} P(t)\mathrm{d}t} \quad (7)$$

Low τ_{rec} = large plasma-wall interaction (similar to $L_{c,w}$)

...BUT we can extend the calculation $\sim$ 10cm inside the plasma!

[5] G.Spizzo, M.Agostini, P.Scarin et al., Nuclear Fusion **57**, 126055 (2017)

Conclusions and Future Perspectives

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- Chaos theory is an important tool to understand particle transport in toroidally confined plasmas for nuclear fusion research

[6] G.Spizzo, R. White and S. Cappello, Physics of Plasmas , 102310 (2007)
[7] L.Marrelli et al, Nuclear Fusion **59**, 076027 (2019)

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- Chaos anyway not far from the Chirikov stochastic threshold: analysis of Poincaré recurrence times show clear signs of dynamical traps, consistent with the subdiffusive character of transport found in RFX plasmas[6]
- This information has been used for the upgrade of the RFX-mod device in Padova, Italy: a modified front-end system will allow for smaller $m = 0$ modes and a weaker phase-locking of $m = 1$ modes[7].

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[7] L.Marrelli et al, Nuclear Fusion **59**, 076027 (2019)