Ambipolar Edge Electric Field with Energy Dependence

G. Spizzo[1] R. B. White[2] M. Agostini[1] G. Ciaccio[1] P. Scarin[1]
O. Schmitz[3] N. Vianello[1]

October 25, 2012

[1] *Consorzio RFX, Padova,Italy*

[2] *PPPL,P.O.Box 451, Princeton,NJ 08543*

[3] *Forschungszentrum Jülich GmbH*
Association EURATOM-FZJ, Jülich, Germany

Outline

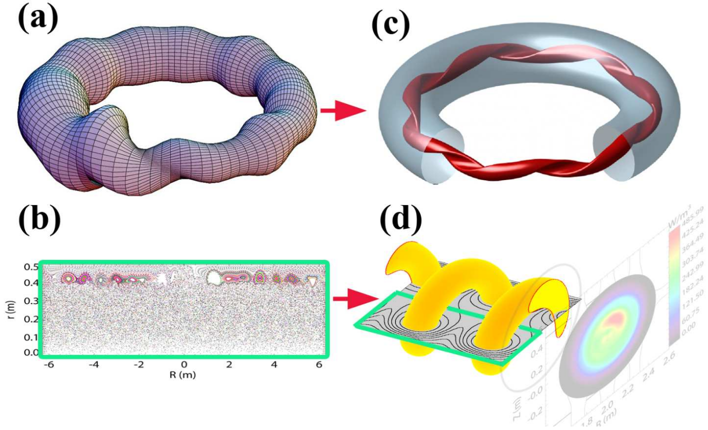
1. Magnetic islands in the edge plasma cause a differential radial diffusion of electrons and ions
2. The resulting electric fields (determined by the ambipolar constraint) can influence the flow and, more generally, plasma performances (e.g. the Greenwald limit [1])
3. The theory and data we present for this sheet potential could be of interest for explaining the restriction of the collisionality window for the application of the resonant magnetic perturbations (RMPs) in Tokamaks [2], and the issue of edge islands interacting with the bootstrap current in stellarators [3];
4. From a theoretical point of view, the problem is how a perturbed toroidal flux of the form $\psi = \psi^0 + \tilde{\psi} \sin u$ ($u = m\theta - n\zeta + \phi_{m,n}$ helical angle) gives rise to an ambipolar potential $\Phi = \tilde{\Phi} \sin u$.

Background

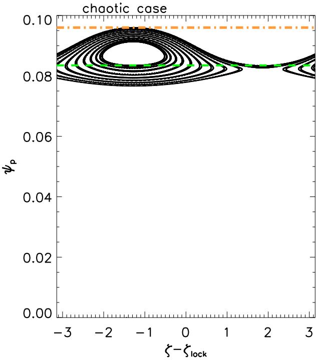
The Reversed-field pinch (RFP) shows a bifurcation from a chaotic regime to *helical equilibrium*, Lorenzini *et al.*, Nat. Phys. 2009; Cappello S. *et al* 2011 NF **51** 103012

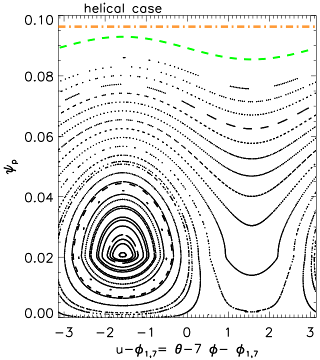
Chaotic regime = MH = dominated by the $m = 0, n = 1$ island

Helical regime = QSH = dominated by the $m = 1, n = 7$ island



Two topologies unified by u





$m = 0$, $n = 1$ Hamiltonian:

$$H(\psi_p, \zeta) = \int q\,\mathrm{d}\psi_p + I\alpha_{0,1}(\psi_p)\sin u$$

with I, g covariant components of $\vec{B}$

helical angle $u = -\zeta + \omega_{0,1} t$

Helical Hamiltonian [a]:

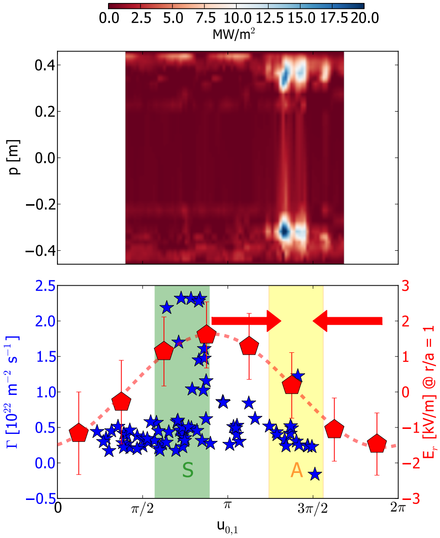
$$\bar{H}(\psi_p, u) = \chi - (g + 7I)\alpha_{1,7}(\psi_p)\sin u$$

with $\chi = \psi_p - 7\psi$ and $u = \theta - 7\zeta + \omega_{m,n} t$

[a] G.Ciaccio, *Bull.Am.Phys.Soc.* **56**, 46 (2011)

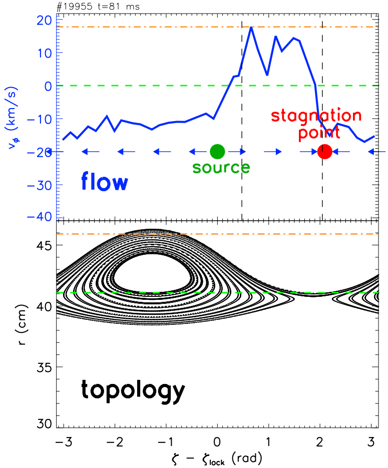
The (0,1) Topology: Measurements

CONSORZIO RFX
Ricerca Formazione Innovazione



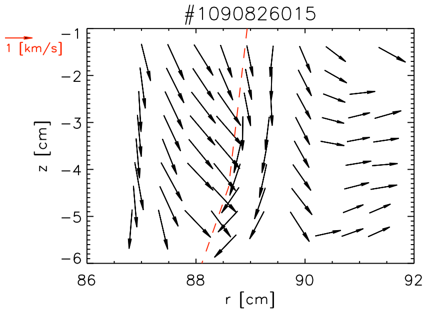
- Approaching $n/n_G \approx 1$ a radiative collapse is due to appearance of localized, poloidally simmetric regions of enhanced radiation [1]
- Particles coming from the source (**S**) are **convected from both sides** towards a stagnation (accumulation) point (**A**)

The (0,1) Topology: Measurements



- **Two null points** of flow define a convective cell
- The convective cell has the same symmetry of the $(m = 0, n = 1)$ mode resonating at $q = 0$
- in particular, the stagnation point corresponds to the X-point (XP) of the island

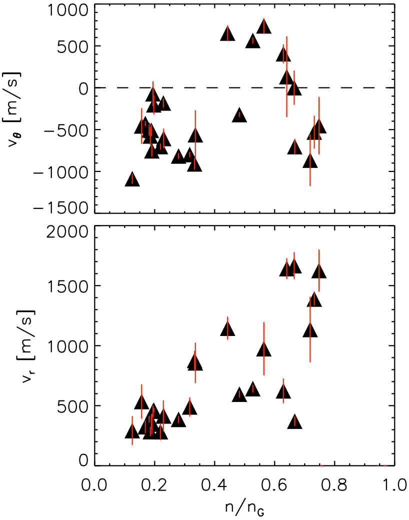
Null points of v_θ: C-mod



- At high density ($n/n_G \approx 0.6$) in the C-mod tokamak, v_θ shows a null point located at the separatrix
- $v_\theta < 0$ (ion diamagnetic drift direction) inside the separatrix, $v_\theta > 0$ outside [4]
- see also poster JP8.00090 by S.Zweben, this poster session

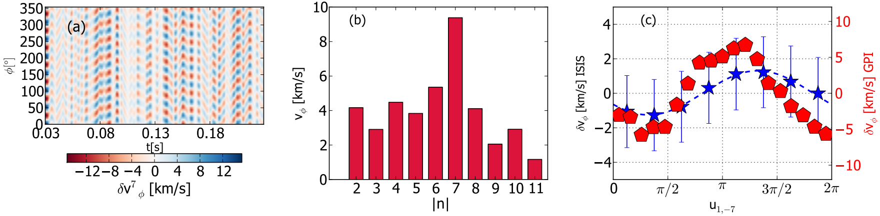
Collaboration between RFX and C-mod (M.Agostini, P.Scarin and S.Zweben)

Null points of v_θ: C-mod



- As a function of density, $v_\theta > 0$ when $n/n_G > 0.4$
- issue with Tokamaks: often the analysis is based on single-point measurements vs time
- the definition of a proper helical angle u translates time in space

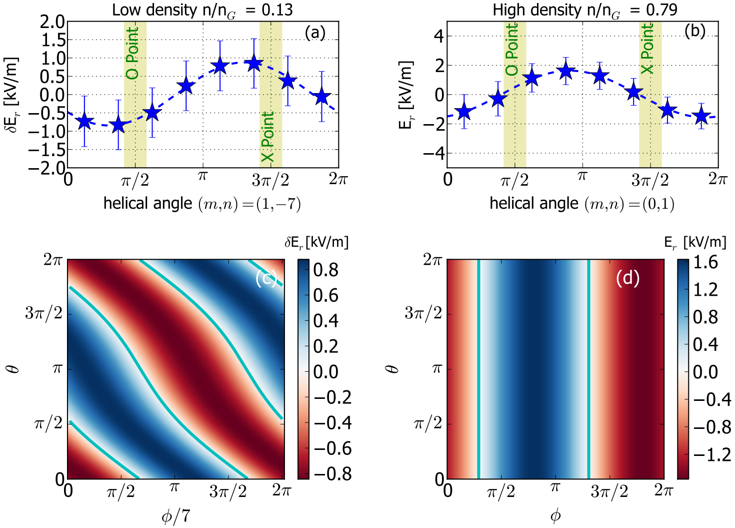
The (1,7) Topology



- Measurements with the array of internal sensors, ISIS [1] show a marked helical symmetry of floating potential and flow
- (a) Toroidal map of toroidal flow at $r = a$ reconstructed through correlation between adjacent pins
- (b) spectrum of flow
- (c) δv_ϕ as a function of u; comparison with GPI [2] data

[1] N.Vianello *et al.*, in Proc. 24th Fus. Energy Conf. (IAEA 2012), EX/p8-02
[2] M.Agostini *et al.*, Plasma Phys. Control. Fusion **51**, 105003 (2009)

(0,1) and (1,7) topologies

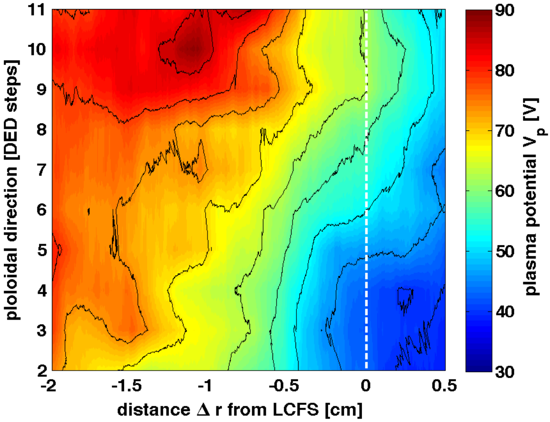


The edge E^r responds with a ripple consistent with the applied helicity

Ripple of E^r in TEXTOR

CONSORZIO RFX
Ricerca Formazione Innovazione

OP $\longrightarrow$



- Measurements of plasma potential inside a $m/n = 4/1$ island induced by means of RMP in TEXTOR
- Excess of V_p towards the island O-point (OP) and decrease (**potential well**) towards the XP [a]

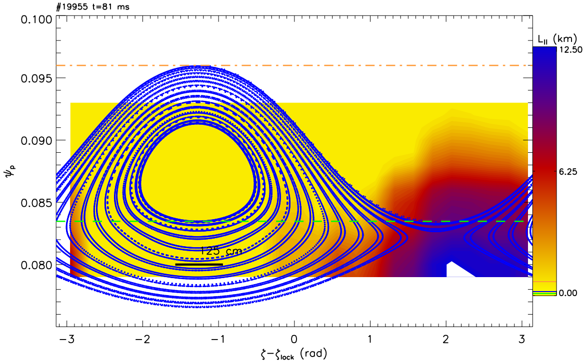
[a] credit: Oliver Schmitz

$\longleftarrow$ **XP**

Test-particle simulations

- We use the guiding-center code Orbit [5] to analyze the magnetic field topology and the motion of (monoenergetic) electrons and ions embedded in the magnetic field (no potential)
- We use as input the eigenfunctions [6] obtained by solving the Newcomb's equations (constraint=magnetic fluctuations measured in the experiment)
- Pitch-angle scattering is implemented by taking into account ion-ion, ion-electron, electron-electron, electron-ion, and ion-impurity pitch-angle scattering, using the Kuo-Boozer approach [7]
- The standard Orbit perfectly absorbing wall has been modified [8] to take into account **recycling** $R = 1$

Simulations - Connection lengths



Parallel electron connection length to the wall can be defined as

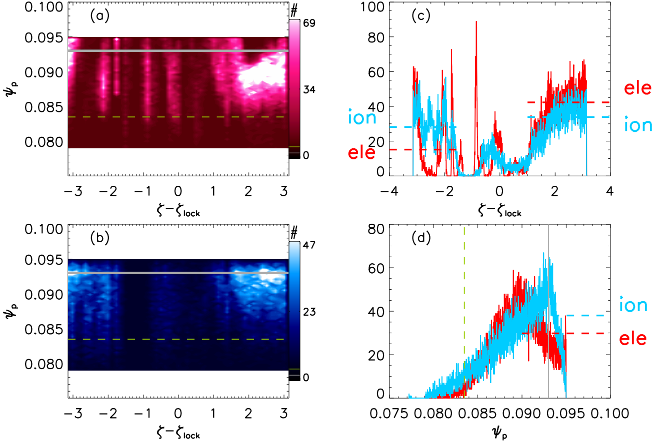
$L_{||}(\psi_p, \theta, \zeta) \simeq v_{th}\tau_{trav}$ [9]

τ_{trav} electron travel time between $(\psi_{p,1}, \theta_1, \zeta_1)$ and $(\psi_{p,2}, \theta_2, \zeta_2)$ - evaluated by Orbit

Initial and final conditions are $(0.093, \textit{random}, \zeta_1)$ and $(\psi_{p,2}, \textit{random}, \textit{random})$

Distinction between a "laminar" and "ergodic" zone, along the toroidal angle ζ

Simulations - diffusion plots



Electrons spend on average more time near the XP than ions do, and less time near the OP (**ions =larger drifts**)

TEXTOR collaboration

The collaboration aims at:

- finding a link between some of the phenomena seen in the tokamak RMP's, such as the density "pump out" and the change of sign of the edge electric field, and the convective cells seen in the RFP edge, in the helical and MH cases
- help to understand if there is a unified picture of the density limit in RFPs and tokamaks;
- make clear the role of the wall, in particular the recycling behavior of the first wall.

Orbit has been adapted to the equilibrium of TEXTOR, and a proper form for the radial perturbation has been developed, on the basis of the analytical formulae used in TEXTOR, which are given e.g. in [10]. The resulting Poincaré plots (red=electrons, black=ions) for the modes (m=12,n=4) and (m=3,n=1) have been produced.

m/n=12/4 (run=95896)

Figure : Field lines

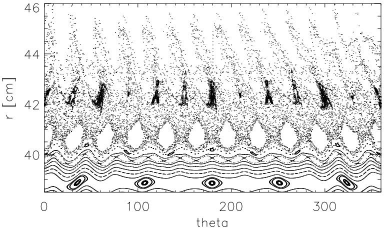


Figure : 50 *eV*

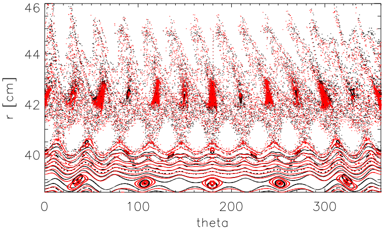


Figure : 100 *eV*

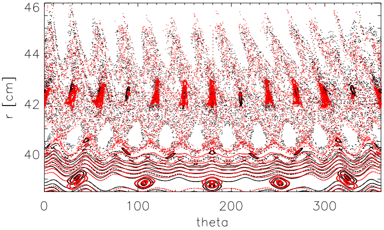
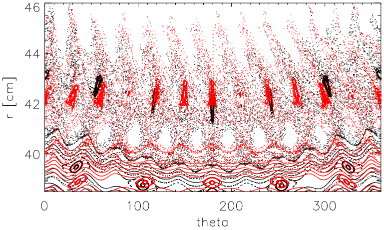


Figure : 1 *keV*



m/n=3/1 (run=109269)

Figure : Field lines

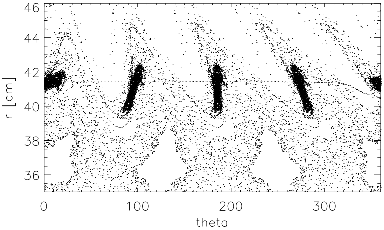


Figure : 50 *eV*

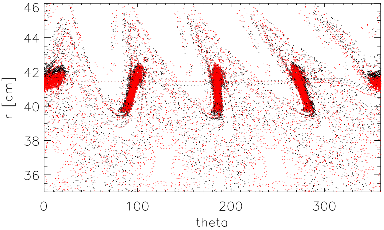


Figure : 100 *eV*

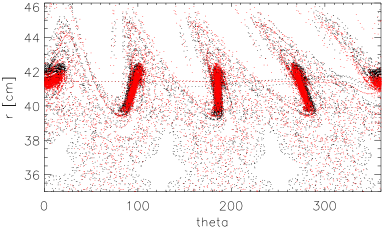
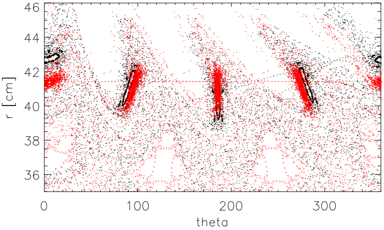


Figure : 1 *keV*



The ambipolar mechanism

The picture of electrons free-streaming to the wall is naif $\rightarrow$ a strong potential builds up to balance radial diffusion

- Since ions diffuse only a few Larmor radii from the deposition surface, the potential is determined mainly by electrons following closely the field lines
- as a rule of thumb, increased electron mobility in the laminar zone w/ recycling wall decreases E_r (more negative E_r)
- . . . therefore, quasi-neutrality is reached at the expense of the symmetry in the toroidal angle
- the potential island is parent to the magnetic $(m = 0, n = 1)$ island
- Details of the model for the potential in Ref. [8]

The (0,1) potential - simulations

Angular dependence derived from data (GPI+ISIS):

$$\mathcal{A}(\zeta) = 2e^{-(\zeta-\zeta_0)^2/2\sigma_\zeta^2} - 1$$

such that

$$\Phi(\psi_p, \zeta) = -E_a\psi_p + V(\psi_p) \times \mathcal{A}(\zeta)$$

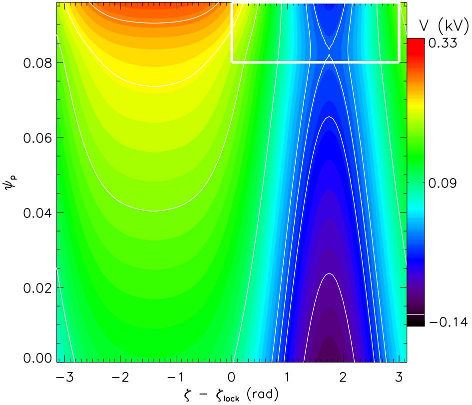
and $E^{\psi_p} = -V'(\psi_p) + E_a$, with

$$E^{\psi_p} = E_a + \frac{1}{2} E_{r,w} \times \left[\tanh\left(\frac{\psi_p - \psi_{p,rv}}{\sigma_{\psi_p}}\right) + 1\right]$$

Assume $\frac{\partial\Phi}{\partial\theta} = 0$

free parameters are $E_{r,w}$ and ζ_0

Potential $\Phi(\psi_p, \zeta)$



Convective cell in the edge

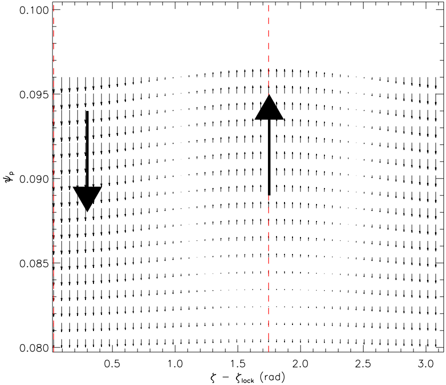
CONSORZIO RFX
Ricerca Formazione Innovazione

As a consequence of $\frac{\partial \Phi}{\partial \zeta} \neq 0$, a convective cell forms near the edge

Motion across the potential $\vec{v} \cdot \nabla \Phi = 0$ (on the equipotential surfaces)

$\rightarrow$conserves kinetic energy

Field E^{ψ_p}, E^{ζ}



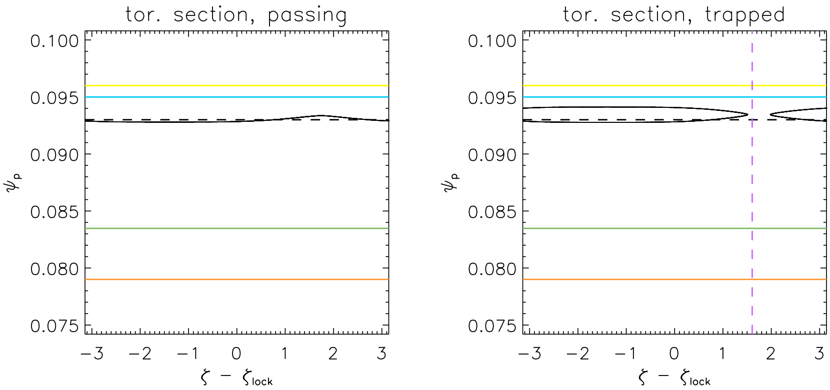
More on the convective cell

CONSORZIO RFX
Ricerca Formazione Innovazione

The convective cell is linked to the fact $\frac{\partial \Phi}{\partial \zeta} \neq 0$

- convective motion $\vec{v} \cdot \nabla \Phi = 0$
- this means
$$\frac{\partial \Phi}{\partial \psi} \vec{v} \cdot \nabla \psi + \frac{\partial \Phi}{\partial \zeta} \vec{v} \cdot \nabla \zeta = 0$$
- Usually, the velocity is a flux function since electrons rapidly equilibrate the potential on the flux surface ψ
- This is no more true if $\frac{\partial \Phi}{\partial \zeta} \neq 0$

Determine the potential amplitude



- Vary the free parameter $E_{r,w}$ until electrons are trapped (no perturbations)
- Creation of a new whole population of trapped particles, opposing parallel streaming of electrons to/from the edge
- Necessary condition for trapping $\rightarrow T_e \leq e\Phi_{hill}$

Determine the potential amplitude

CONSORZIO RFX
Ricerca Formazione Innovazione

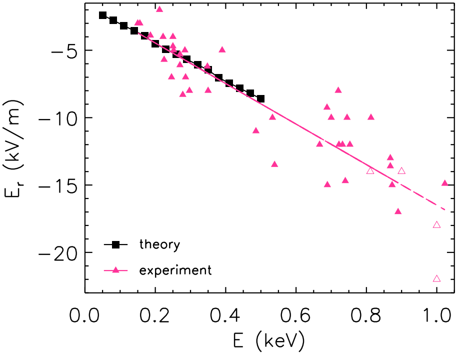
Linear dependence with energy,
$E_{r,w} = -T_e/eL_{well}$

$L_{well} \sim 6.6$ cm ($L_{well} = 2\sigma_{\psi_p}$)

Experimental evaluation from GPI

$$E_r = \frac{T_i}{Ze}\frac{\nabla P_i}{P_i} + v_\phi B_\theta$$
$$\simeq 0.15 E_r + v_\phi B_\theta$$

Field is likely to be ambipolar

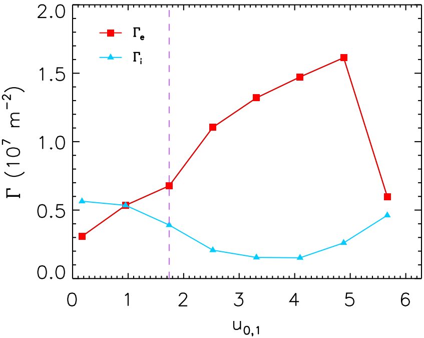


Determine the potential phase

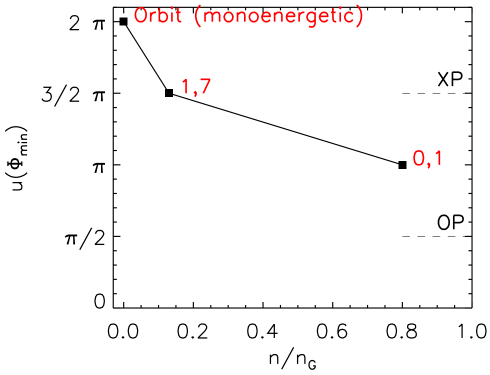
CONSORZIO RFX
Ricerca Formazione Innovazione

Vary the free parameter ζ_0 ($\equiv$ helical angle u) until Orbit balances the fluxes $\Gamma_e = \Gamma_i$ at $\psi_p = 0.079$

- Problem with Orbit: ambipolarity is reached when $u \simeq 2\pi$
- in high density RFX discharges dominated by the (0,1) mode $u \sim \pi$ (see slide 8,(b)-(d))
- in low density discharges dominated by the (1,7) mode $u \sim 3/2\pi$ see slide 8,(a)-(c))



The monoenergetic potential- summary



- The phase lag between potential and island depends on n/n_G
- It is likely that, given the obvious link between particle energy (=collisions) and trapping in the potential (slide 21), the dependence on collisionality is wrapped up in the parameter n/n_G

Is the whole Greenwald limit, expressed heuristically as a function of n_G [1], a problem of collisional trapping in a potential?

Collisions w/ energy

Coulomb collisions modify the direction of particle velocity: this has to be translated in the frame of reference of the guiding center, in terms of the particle pitch $\lambda = v_{||}/v$ and collision frequency ν

$$\lambda_{k+1} = \lambda_k \cdot (1 - \nu \mathrm{d}t) \pm \sqrt{(1 - \lambda_k^2)\, \nu \mathrm{d}t} \qquad (1)$$

This is called pitch angle scattering, and is the MonteCarlo scattering operator that allows Orbit for exchanging momentum between particles (bananas and neoclassical effects).
The new task is to introduce energy exchanging collisions, following Eqs.(61)-(63) of Ref. [7]

BK operator

CONSORZIO RFX
Ricerca Formazione Innovazione

NRL 2011, p.31: the drag frequency of a species α against β ("energy loss") is

$$\nu_E^{\alpha/\beta} = -2\left[\frac{m_\alpha}{m_\beta}\psi(x) - \psi'(x)\right]\nu_0^{\alpha/\beta} = -g(x)\nu_0^{\alpha/\beta} \qquad (2)$$

where $\nu_0^{\alpha/\beta} \sim n_\beta / v_\alpha^3 \log\Lambda$, $x = x^{\alpha/\beta} = v_\alpha^2/v_\beta^2$ and ψ is the Rosenbluth potential

$$\psi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{d}t\, t^{1/2} e^{-t} \qquad (3)$$

The corresponding MonteCarlo operator is, according to [7]

$$E_{k+1} = E_k - (2\nu_E \mathrm{d}t)\left[1 - \frac{m_\alpha}{m_\beta}\frac{g'}{g}\right] \pm 2\sqrt{E_k T_i(\nu_E \mathrm{d}t)} \qquad (4)$$

BK operator-corrected

CONSORZIO RFX
Ricerca Formazione Innovazione

... But this has not quite the right asymptotics!
In fact, from Eq. (4) and $m_\alpha = m_\beta$ (ion-ion scattering) we obtain

$$\frac{\mathrm{d}E}{E} = \frac{E_{k+1} - E_k}{E_k} \propto -x^{-3/2}\left[g(x) - g'(x)\right], \tag{5}$$

which has the asymptotic behavior

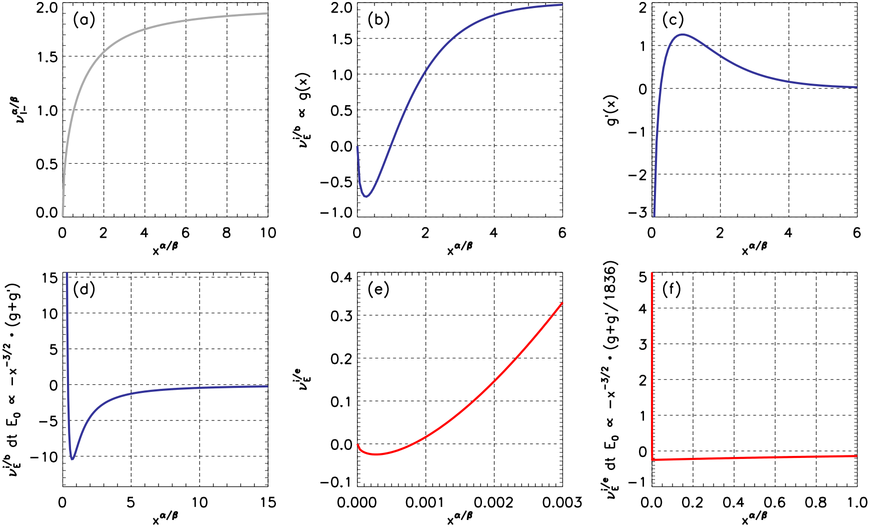
$$\begin{aligned} -x^{-3/2}\left[g(x) - g'(x)\right] &\xrightarrow{x=0} -\infty \\ -x^{-3/2}\left[g(x) - g'(x)\right] &\xrightarrow{x=+\infty} -2x^{-3/2} \to 0 \end{aligned} \tag{6}$$

In particular, the $-\infty$ divergence in zero causes low-energy particles to be cooled down, which makes no sense. The operator has to be modified as

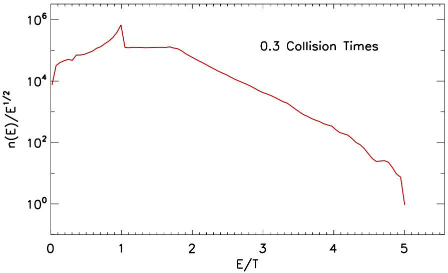
$$E_{k+1} = E_k - (\nu_E \mathrm{d}t)\left[1 + \frac{m_\alpha}{m_\beta}\frac{g'}{g}\right] \pm \sqrt{2E_k T_i(\nu_E \mathrm{d}t)} \tag{7}$$

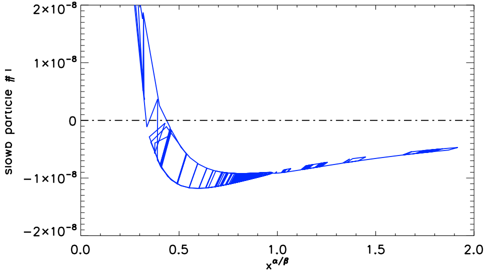
which has the right asymptotics and converges to the NRL result (2) if one drops g'.

Rosenbluth potentials



Maxwellian





Relaxation to a Maxwellian: single particle, initial $E = 400$ eV, ion background $T_i = 230$ eV, 0.3 collisional times (analogue to Fig.4 in [7])

Maxwellian = straight line in the plot

Bibliography

This Beamer template was created by

Alessandro Scaggion

[1] M.E.Puiatti, *et al*
2009 *Phys. Plasmas* **16** 012505

[2] H. Stoschus, O. Schmitz, H. Frerichs, *et al.*
Nucl. Fusion **52**, 083002 (2012).

[3] Y. Feng, *et al.*
Plasma Phys. Control. Fusion **53**, 024009 (2011)

[4] M.Agostini *et al.*
Nucl. Fusion **51** (2011) 053020

[5] Roscoe B. White and Morrell S. Chance
Phys. Fluids **27**, 2455-2467 (October 1984).

[6] P. Zanca and D. Terranova
Plasma Phys. Control. Fusion **46**, 1115 (2004).

[7] Allen H. Boozer and Gioietta Kuo-Petravic
Phys. Fluids **24** (1981) 851.

[8] G.Spizzo, *et al.*
2012 *Nucl. Fusion* **52** 054015

[9] P.Scarin, *et al.*
2011 *Nucl. Fusion* **51** 073002

[10] Finken K.H. *et al.*
2005 The Structure of Magnetic Field in the TEXTOR-DED: Schriften des Forschungszentrums Jülich,Band 45